Journal of Scientific Research
Banaras Hindu University, Varanasi

Vol. 61, 2017 : 179-183
ISSN : 0447-9483

# A NEW LOG-PRODUCT-TYPE ESTIMATOR USING AUXILIARY INFORMATION

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**Abstract :**

In this paper, we proposed some new type estimators for estimating finite population mean of study variable. The mean square error of the proposed estimators have been obtained up to the first order of approximation. Here we shown that our proposed estimators are more efficient than usual estimator. An empirical study is also carried out to demonstrate the performance of proposed estimators.

**Keywords**: study variable, auxiliary variable, population mean, mean square error.

## 1. Introduction:

The problem of estimating the finite population mean in the presence of an auxiliary variable has been widely discussed in the finite population sampling literature. It is well established phenomenon in the theory of sample surveys that the auxiliary information is often used to improve the accuracy of estimators of unknown population parameters. The use of auxiliary information at the estimation stage appears to have started with the work of Watson(1937) and Cochran(1940). It is well established that when the auxiliary information is to be used at the estimation stage, the ratio, product and regression methods of estimation are widely used in many situations. Many authors have used the auxiliary information for increasing the precision of estimators in estimating the population mean of study variable.

Let U= ( $U_1$, ……, $U_N$ ) be a finite population of N units from which we draw a sample of size n. Let the value of the study variable Y and the auxiliary variable X for the $i^{th}$ unit (i=1,2,….,N) of the population be denoted by $Y_i$ and $X_i$ and for $i^{th}$ unit in the sample (i=1,…n) by $y_i$ and $x_i$ respectively.

For the sample observations we have,

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \quad \text{and} \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \ .$$

For the population observations we have,

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i \quad \bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i \ , \ S_Y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2 \ , \ S_X^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2$$

and $S_{XY} = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \overline{Y}\right)\left(X_i - \overline{X}\right)$ .

Let us define,

$\overline{y} = \overline{Y}(1+\varepsilon_\circ)$ and $\overline{x} = \overline{X}(1+\varepsilon_1)$ such that $E(\varepsilon_\circ) = E(\varepsilon_1) = 0$ and under SRSWOR sampling scheme-

$E(\varepsilon_\circ^2) = \theta C_Y^2$ , $E(\varepsilon_1^2) = \theta C_X^2$ , and $E(\varepsilon_\circ\varepsilon_1) = \theta\rho C_X C_Y$ .

where $\theta = \left(\frac{1}{n} - \frac{1}{N}\right)$ , $C_Y^2 = \frac{S_Y^2}{\overline{Y}^2}$ , $C_X^2 = \frac{S_X^2}{\overline{X}^2}$ and $\rho = \frac{S_{XY}}{S_X S_Y}$ .

**Estimators in literature**:

1. The variance of usual estimator $\overline{y}$ is given by,

$$Var(\overline{y}) = \overline{Y}^2 \theta C_y^2 \tag{1}$$

2. The linear regression estimator $\overline{y}_{lr}$ is given by,

$$\overline{y}_{lr} = \overline{y} + k\left(\overline{X} - \overline{x}\right)$$

The optimum value of k at which variance is minimum is given by

$$k_{opt} = \rho\frac{C_y}{C_x}$$

The minimum variance of the estimator $\overline{y}_{lr}$ is given by,

$$Var(\overline{y}_{lr})_{min.} = \overline{Y}^2 \theta C_y^2 \left(1-\rho^2\right) \tag{2}$$

**2. Proposed Estimator**:

In this section we propose a new log type estimator as-

**1.** $\overline{y}_{pr} = \overline{y} + \alpha \log\left(\frac{\overline{x}}{\overline{X}}\right)$

Expanding $\overline{y}_{pr}$ in terms of $\varepsilon_\circ$ and $\varepsilon_1$ and then taking expectations up to the first order of approximation, we get the mean square error (MSE) of the estimator $\overline{y}_{pr}$,

$$MSE(\bar{y}_{pr}) = \bar{Y}^2\theta C_y^2 + \alpha^2\theta C_x^2 + 2\bar{Y}\alpha\theta\rho C_y C_x \quad (3)$$

The optimum value of $\alpha$ at which MSE is minimum is given by

$$\alpha_{opt.} = -\left(\frac{\bar{Y}\rho C_y}{C_x}\right)$$

The min. MSE of the estimate $\bar{y}_{pr}$ is given by

$$MSE(\bar{y}_{pr})_{min.} = \bar{Y}^2\theta C_y^2\left(1-\rho^2\right) \quad (4)$$

We propose another estimator $\bar{y}_p$ as

**2.** $\bar{y}_p = \bar{y}(w_1+1) + w_2 \log\left(\frac{\bar{x}}{\bar{X}}\right)$

Expanding $\bar{y}_p$ in terms of $\varepsilon_\circ$ and $\varepsilon_1$ and taking expectations up to the first order of approximation, we get the mean square error(MSE),

$$MSE(\bar{y}_p) = \bar{Y}^2\theta C_y^2 + W_1^2 A + W_2^2 B + 2W_1 C + 2W_2 D + 2W_1 W_2 E \quad (5)$$

where,

$$A = \bar{Y}^2\left(1+\theta C_y^2\right),\ B = \theta C_x^2,\ C = \bar{Y}^2\theta C_y^2,$$

$$D = \bar{Y}\theta\rho C_y C_x,\ E = \bar{Y}\theta\left(\rho C_y C_x - \frac{C_x^2}{2}\right).$$

The optimum values of $W_1$ and $W_2$ at which MSE is minimum is given by

$$W_{1opt} = \frac{(CB-DE)}{\left(E^2-AB\right)} \quad \text{and} \quad W_{2opt} = \frac{(AD-CE)}{\left(E^2-AB\right)}$$

The minimum MSE of the estimator $\bar{y}_p$ after substituting the optimum value of $W_1$ and $W_2$, is given

by

$$MSE(\bar{y}_p)_{min.} = C + \left(\frac{BC^2 + AD^2 - 2CDE}{E^2 - AB}\right) \quad (6)$$

## 3. Efficiency Comparison

In this section, we compare the efficiency of the proposed estimators under optimum condition with existing estimators.

From equations (1), (2), (4) and (6) we note that

1. $\mathrm{Var}(\bar{y}) - \mathrm{MSE}(\bar{y}_{pr})_{\min.} \geq 0$ if $\rho^2 \geq 0$

2. $\mathrm{Var}(\bar{y}) - \mathrm{MSE}(\bar{y}_{p})_{\min.} \geq 0$ if $\left(\dfrac{BC^2 + AD^2 - 2CDE}{E^2 - AB}\right) \leq 0$

3. $\mathrm{Var}(\bar{y}_{lr}) - \mathrm{MSE}(\bar{y}_{p})_{\min.} \geq 0$ if $\left(\dfrac{BC^2 + AD^2 - 2CDE}{E^2 - AB}\right) \leq 0$

## 4. Empirical Study

For empirical study we consider the following data sets.

**Population 1. [ Source: Murthy (1967, p.228)]**

X: Fixed Capital, Y: Output,

$$N = 80, n = 20, \bar{Y} = 51.8264, C_y = 0.3542, C_x = 0.7507, \rho = 0.9413.$$

**Population 2. [ Source: Das (1988)]**

X: Number of agricultural laborers for 1961, Y: Number of agricultural laborers for 1971,

$$N = 278, n = 30, \bar{Y} = 39.0680, C_y = 1.4451, C_x = 1.6198, \rho = 0.7213.$$

The Percent Relative Efficiency (PRE) of an estimator is defined as

$$\mathrm{PRE}(\bullet) = \frac{\mathrm{Var}(\bar{y})}{\mathrm{MSE}(\bullet)} \times 100$$

**Table 1**: PRE (percent relative efficiency) of different estimators with respect to usual unbiased estimator .

| **Estimators** | **Population 1** | **Population 2** |
|---|---|---|
| $\bar{y}$ | 100.00 | 100.00 |
| $\bar{y}_{lr}$ | 877.5447 | 208.4522 |
| $\bar{y}_{pr}$ | 877.5447 | 208.4522 |
| $\bar{y}_{p}$ | 924.4771 | 230.4309 |

## 5. Conclusion

From Table 1, we observe that our proposed estimator $\bar{y}_{pr}$ is equally efficient to usual linear regression estimator $\bar{y}_{lr}$ and second proposed estimator $\bar{y}_{p}$ is more efficient than usual linear regression estimator $\bar{y}_{lr}$ .

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